Supervision 21
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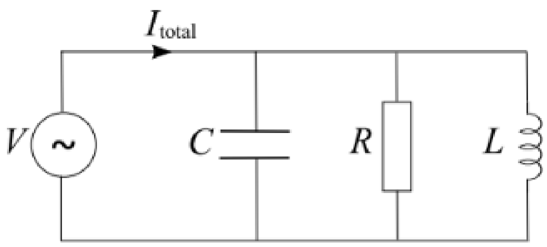
In our last supervision, We will discuss the long questions in Section B in Cavendish and Christ's College mock exams. Per your request, we will use the following questions from past exams to revise the following contents.

Oscillations in electrical circuits
Application of zero-momentum frames in Newtonian and Relativistic collisions
Electrical and magnetic fields
Special relativity
Phasor diagram in optical interference, e.g., double-slit experiments

1. Oscillations in electrical circuits

C11 in 2016 Physics 1A exam

C11 Deduce an expression for the complex impedance of an inductor, assuming that for an inductive element $V = L\,\mathrm{d}I/\mathrm{d}t$ (where I is the current through the element and V is the voltage across it). [2]



Consider the circuit in the diagram. A sinusoidal voltage of angular frequency ω, $V = V_0 \cos(\omega t)$, is applied to the circuit. Find a general expression for I_{total}, the current drawn from the voltage source. Sketch the amplitude and phase of the current as a function of the angular frequency. [6]

The condition of resonance is that I_{total} is in phase with the applied voltage. Calculate the resonant angular frequency ω_{res}. Does the amplitude of I_{total} take a maximum or minimum value at this frequency? Explain what is happening in the circuit at resonance. [3]

Consider the ratio of the amplitude of the current through the resistor to the amplitude of the total current drawn from the voltage source. We define the bandwidth $\delta\omega$ of this circuit to be the angular frequency interval over which this ratio is greater than $1/\sqrt{2}$. Work out an expression for the bandwidth in terms of the values of the components in the circuit. [2]

The sharpness of this resonance may be quantified by a quality factor $Q = \omega_{\text{res}}/\delta\omega$. Relate Q to L, C, and R. Comment on the form of this expression. [2]

2. Applications of zero-momentum frames

B5 in 2022 Christ's College mock exam (Relativistic energy and momentum)

B8 in 2006 Physics 1A exam (Newtonian energy and momentum)

B8 Explain why, when two particles collide elastically, each particle retains its speed in the zero-momentum frame. What can be said about the final directions of the particles in this frame? [3]

A mass m moves in the 'laboratory' frame, at a speed u which is much less than c. It collides elastically, but not head-on, with an equal mass at rest. Use vector diagrams to show that the angle between the final directions is $\pi/2$. [3]

A mass m with speed u collides head-on but *inelastically* with an equal mass at rest, with half of the kinetic energy in the zero-momentum frame being dissipated. Find the final velocities of both particles in the laboratory frame, and show explicitly that the amount of kinetic energy dissipated is the same in both frames. [5]

Show that, if this inelastic collision is *not* head-on, the largest angle through which the incident particle can be deflected in the laboratory frame is $\pi/4$. [4]

3. Special relativity

B6 in 2023 Cavendish mock exam

4. Phasor diagrams in optical waves and interference

C12 in 2009 Physics 1A exam

C12 Explain Huygens's principle and show by means of *two* examples how it can be applied to problems of wave propagation. [3]

The midpoints of two identical, narrow, vertical slits are positioned at A and C, a distance $2d$ apart. A third, identical slit is positioned at B, midway between A and C. This slit contains an optical element which introduces a phase shift of π to any transmitted light. The 3 slits are illuminated by a plane wave of wavelength λ, which is incident normally on the plane of the slits. The intensity of the resulting wave, $I(\theta)$ is observed at a distant point P. The position vector of P from B subtends an angle θ with the perpendicular bisector of the line AC.

Obtain an expression for $I(\theta)/I(0)$. [6]

Sketch $I(\theta)/I(0)$ marking the angular positions and amplitudes of the important features. [3]

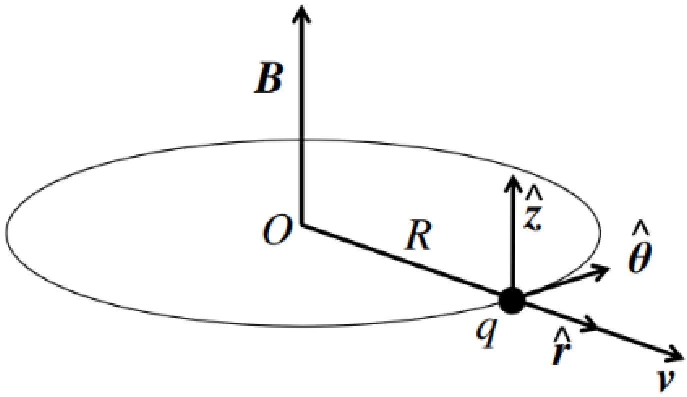
Illustrate your answer with phasor diagrams. [3]

5. Electrical and magnetic forces/fields

E14 in 2016 Physics 1A exam

E14 Define the terms in the Lorentz force law $\boldsymbol{F} = q\,(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$. [1]

Consider a cylindrical coordinate system with an origin O and unit vectors $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$, $\hat{\boldsymbol{z}}$. A circular rubber ring is placed in a static, uniform magnetic field $\boldsymbol{B} = B\hat{\boldsymbol{z}}$. The ring is centred on O and located in the $z = 0$ plane, as shown in the figure below. The ring expands with each part of it having a constant radial velocity $\boldsymbol{v} = v\hat{\boldsymbol{r}}$. A bead carrying a charge q moves around the rubber ring with tangential velocity $\boldsymbol{w} = w\hat{\boldsymbol{\theta}}$.



What are the radial, tangential and axial components of the force $\boldsymbol{F}_{\mathrm{B}}$ exerted by the magnetic field on the bead? [3]

For a ring of radius R, show that the work done by $\boldsymbol{F}_{\mathrm{B}}$ as the bead moves once around the ring is

$$W = -2\pi RqvB.$$

[*You may assume that $w \gg v$ and so the radius does not change significantly during the time it takes for the bead to move once around the ring.*] [1]

What tangential electric field $\boldsymbol{E} = E\hat{\boldsymbol{\theta}}$ would be required to produce a force equivalent to the tangential component of $\boldsymbol{F}_{\mathrm{B}}$? [2]

What is meant by the electromotive force? [1]

Use the expression for the work done as the bead moves around the ring (W) to show that the tangential force on the bead may be considered to be due to an electromotive force ϵ induced in the ring which is given by

$$\epsilon = -\frac{d\phi}{dt},$$

where ϕ is the magnetic flux through the ring. [3]

By considering the radial component of $\boldsymbol{F}_{\mathrm{B}}$, show that the ring is doing work at a rate $-Bqvw$ on the bead. Compare this with the rate at which work is done by the tangential component of $\boldsymbol{F}_{\mathrm{B}}$ and comment on the result. [4]