# Easter Mock for Mock

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This is an Easter-term Physics IA mock exam paper written by Jiachen Jiang. This mock exam is made up of questions from past tripos papers[1].

This mock exam does not cover modules in the Easter term but only Michaelmas and Lent terms. The difficulty and number of the problems in Section A should be comparable to the annual Physics 1A mock exam paper at Cavendish.

Answer the whole of Section A and at least two questions in Sections B and C. For revision purposes, you should try to solve as many questions in Sections B and C as possible. You may use the approved calculator. Please refer to the table of constants in a past exam paper if necessary.

Some of the questions from the ancient exam papers have been modified to match the updated curriculum of Physics IA at Cambridge.

[1] I do not take credit for writing the problems. Contact me for detailed solutions.

## Section A

1. **1997-1998 Cambridge:** The rotor arm of a torsional oscillator in an antique clock consists of a weight of mass $M$ of a tiny size, attached to the end of a rod of mass $m$ and length $a$. Calculate its moment of inertia $I$ relative to the upper end of the rod. If M, m and a are approximately 50g, 10g and 1m, respectively, what is the fractional error in the calculated value of $I$ when a, M, and m are each measured with a random error of 5 per cent?

Solution: $I = Ma^2 + \frac{1}{3}ma^3$

2. **2002-2003 Cambridge:** Calculate the wavelength of an electron of total energy 50 MeV. ($m_\mathrm{e} = 0.511$ MeV c$^{-2}$)

Solution: 25 fm

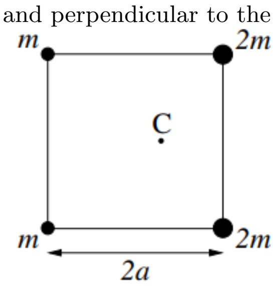
3. **1998-1999 Cambridge:** Estimate the mean distances from the sun of Venus and Mars, as fractions of the Earth's orbit, given that their orbital periods are 0.62 and 1.88 years respectively.

Solutions: $T = 2\pi\sqrt{\frac{a^3}{GM}}$

4. **2003-2004 Cambridge:** A cylindrical hoop rests on a rough uniform incline. It is released and rolls without slipping through a vertical distance $h_0$. It then continues up a perfectly smooth incline. What height does it reach?

Solution: $\frac{1}{2}h_0$

5. **2003-2004 Cambridge:** A rigid frame is constructed of light rods of length $2a$ and a small masses, $m$ and $2m$, are attached to the corners as shown. Find the moment of inertia for rotation about the axis through the centre of mass, $C$, and perpendicular to the plane of the square frame.




Solution: $\frac{34}{3}ma^2$

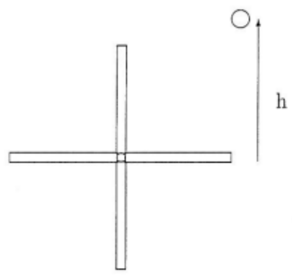
6. **2002-2003 Cambridge** A camera used to photograph documents has a simple lens of focal length of 8.5 cm. How far from the camera lens should a 20 cm by 10 cm diagram be placed if the image on the film is to be 4 cm by 2 cm?

Solution: 51 cm

## Section B

1. **2008-2009 Cambridge:** Define the impulse exerted when a force acts for a short time and relate the impulse to the changes in linear and angular momenta during collisions.

A toy windmill consists of four thin uniform rods of mass $m$ and length $l$ arranged at right angles in a vertical plane, around a thin, fixed horizontal axle about which they can turn freely, as shown in the diagram. Show the moment of inertia of the windmill about the axle is $\frac{4}{3}ml^2$.




Initially, the windmill is stationary. A small ball of mass $m$ is dropped from a height $h$ above the axle, and makes an elastic collision with the end of a horizontal rod. Derive an expression for the angular speed of the windmill after the collision.

To what height does the ball rebound?

Solutions: $\omega = \frac{6}{7}\frac{\sqrt{2gh}}{l}$; The ball will rebound to a height of $\frac{1}{49}h$.

2. **1997-1998 Cambridge:** Two identical solid spheres of radius $a$ and density $\rho$ are in orbit around their centre of mass, far from any other object. If the spheres are just not touching, show that the period of the orbit is given by

$$T = 2\sqrt{\frac{3\pi}{\rho G}}. \tag{1}$$

Find an expression for the gravitational potential in the plane of the orbit at a distance $r(> 2a)$ from the centre of mass when

*a.* the point of observation and the centres of the two spheres are all in a line, and

*b.* a quarter of a period later.

An experiment is concerned with the time variation of the gravitational potential of the system of two spheres. Use the above results to show that in the limit $r \gg a$, the difference between the maximum and minimum in the gravitational potential is given by

$$\frac{3MGa^2}{r^3}. \tag{2}$$

3. **1997-1998 Cambridge:** In the mechanics of special relativity, the expression $E^2 - p^2c^2$ is invariant. Explain what this means, and define the quantities $E$, $p$ and $c$. What is the value of $E^2 - p^2c^2$ for a single particle of mass $m$, and for a system of many particles?

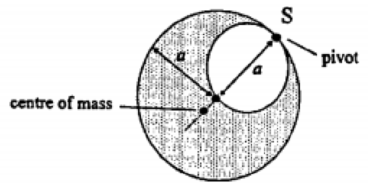
A particle of mass $m$ is travelling through the laboratory at a speed of $\frac{c}{\sqrt{3}}$ and collides head-on with another particle of mass $2m$ travelling at a speed of $v$ in the opposite direction. The two particles combine to form a composite particle of mass $M$, which is stationary. Show that

*a.* $v = c/3$.

*b.* $M = (3 + \sqrt{3})m/\sqrt{2}$.

Why is the mass of the composite particle greater than the sum of the masses of the two colliding particles?

4. **1998-1999 Cambridge:** What is meant by moment of inertia? Find the moment of inertia of a uniform disk of radius $a$ and mass $m$ about an axis perpendicular to its plane and through its centre.




An earring consists of a disk of radius $a$ with a circular piece of radius $a/2$ removed, as shown in the diagram. It is suspended on a pivot at point S and can sing in its own place. By using the theorem of parallel axes, find the moments of inertia about S of complete disks of radii $a$ and $a/2$. Show that the moment of inertia about S of the earring is

$$I_{\rm tot} = \frac{45ma^2}{32}, \tag{3}$$

where $m$ is the mass of the whole disk without the removal of a section.

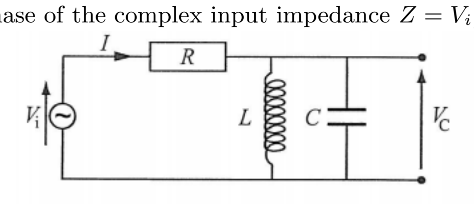
Show that the centre of mass of the earring is as shown on the diagram and is at a distance $a/6$ from the midpoint of the diameter through S.

Show that the angular frequency, $\omega$, for small oscillations about S in the plane of the earring is given by

$$\omega = \sqrt{\frac{28g}{45a}}. \tag{4}$$

## Section C

5. **2009-2010 Cambridge** An electrical circuit consisting of a resistor R in series with a parallel combination of an inductor L and a capacitor C, is connected to a voltage source $V_i$, as shown in the diagram. Calculate the magnitude and phase of the complex input impedance $Z = V_i/I$ of the circuit.




The angular frequency $\omega$ of the sinusoidal input voltage $V_i$ is varied.

Find an expression for $V_C/V_i$ as a function of $\omega$. Make an annotated sketch of the amplitude and phase of the ratio $V_C/V_i$ as a function of $\omega$. What could the circuit be used for?

The circuit with $R = 50\Omega$, $L = 150$mH and $C = 60\mu$F, is now driven by a sinusoidal voltage $V_i(t) = V_0 \cos(2\pi\nu t)$ with $V_0 = 100$V and frequency $\nu = 50$Hz. Calculate the maximum instantaneous currents flowing through each of the three elements $R$, $L$ and $C$.

Calculate the root mean square value of the current I flowing through the circuit.

(Similar to the problem sheet question Q23. $Z_{\text{tot}} = R + \frac{i\omega L}{1-\omega^2 LC}$)

6. **Oxford Prelims:** High-energy photons propagating through space can convert into electron-positron pairs by scattering with cosmic microwave background (CMB) photons via the following process.

$$\gamma_1 + \gamma_2 \rightarrow e^+ + e^- \tag{5}$$

Taking the average CMB temperature of 2.8 K, a typical CMB photon will have an energy of roughly $E_2 = 7 \times 10^{-4}$ eV. The high-energy photons have a much higher energy compared to the CMB photons. Calculate the minimum energy $E_1$ required for the high-energy photon to produce an electron-positron pair ($m_\mathrm{e} = 511$ keV) if

*a.* the CMB photon momentum is perpendicular to that of the high-energy photon.

*b.* the CMB photon propagates in the direction opposite the high-energy photon.

*c.* the CMB photon propagates in any direction with an angle $\phi$ relative to the high-energy photon.

*d.* Suppose the CMB photon propagates in the same direction as the high-energy photon. Based on your answer to question *c*, is it ever possible for the two photons to collide and produce an electron-positron pair?

Solution *c.*: $\frac{2m_\mathrm{e}^2c^4}{[1-\cos(\phi)E_2]}$

7. **2004-2005 Cambridge:** The transverse displacement $y$, for waves travelling in the $x$ direction on string under tension $T$, is given by

$$y = y_0 \cos(\omega t - kx) \tag{6}$$

where $\omega$ and $k$ are both positive. Sketch the displacement pattern at time $t = 0$ and $t = \pi/2\omega$.

Explain why this describes the motion of a wave travelling in the positive $x$ direction, show that the velocity of the wave $c$ is equal to $\omega/k$.

Sketch a labelled graph of the transverse velocity of string $\partial y/\partial t$ and the gradient of the string $\partial y/\partial x$ against time, at $x = 0$.

Show that the instantaneous rate of work done by the string to the left $x = 0$ on the string to the right is given by

$$W = -T\frac{\partial y}{\partial x}\frac{\partial y}{\partial t} = Ty_0^2 k\omega \sin^2(\omega t). \tag{7}$$

Show that this result is consistent with the expression $\frac{1}{2}y_0^2 k^2 T$ for the mean energy per unit length in the wave.

8. **2008-2009 Cambridge** The Lorentz transformation can be written in the form

$$x' = \gamma(x - vt) \tag{8}$$

$$t' = \gamma(t - \frac{vx}{c^2}) \tag{9}$$

Define all the terms in these equations.

Explain what is meant by time dilation and length contraction in special relativity.

Two spaceships, A and B, leave Earth simultaneously, travelling in opposite directions, with constant speeds $4c/5$ and $3c/5$ respectively, in the Earth's frame S. Each carries a clock synchronised before launching with a clock on Earth. Ship A reaches a star when its clock reads 3 years after launch. It immediately turns around and returns to Earth at speed $4c/5$ in S. As it turns around, it sends a radio message to ship B. Once ship B receives the radio signal, it turns around and returns to the Earth with a speed of $3c/5$.

How far is the star from Earth?

What is the elapsed time, measured by the clock on ship B, when it receives the signal from ship A?

What is the elapsed time on the Earth's clock when ship B reaches Earth after its journey?

Solutions: 4 light years; 18 years; 45 years